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Laminar Boundary Layer with Fluid Injection

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This paper examines the distribution of p1operties through the bounda1y layer, and in particular the temperature, instead of heat transfer at the wall and skin friction which are the usual objects of inte1est The simplifying assumption of P1andtl, Lewis, and Schmidt numbers of 1 yields a solution with interesting results pertaining to boundary-layer cooling Cooling through the bounda1y laye1 is 1educed to a relatively simple function of mass of fluid coolant entering the layer and its specific heat An injectant with a very high specific heat can lower the maximum temperature to about ¼ its normal level for air

Nomenclature

x,y = geometric coordinates—distance along and normal to surface, respectively
p,T = thermodynamic properties of pressure, temperature
ρ = density
h = enthalpy
u = velocity in x direction
v = velocity in y direction
c = mass fraction
C_p = specific heat at constant pressure
μ = viscosity
k = thermal conductivity
Pr = Prandtl number $\mu Cp/k$
Re = Reynolds number $\rho ux/\mu$
Le = Lewis number $CpD\rho/k$
D = binary diffusion coefficient
δ = thickness of boundary layer
Sc = Schmidt number $\mu/\rho D = (Pr/Le)$
$\dot{m}$ = mass flow per unit area
C_f = local coefficient of friction $\tau_w/\frac{1}{2}\rho_\infty u_\infty^2$
τ_w = shear at the wall $\tau_w = \mu_w(\partial u/\partial y)_w$
C_1^* = C_1 for Cp^2T/ho_∞ a maximum
C' = Chapman and Rubesin constant from $\mu/\mu_\infty = C'T/T_\infty$

Subscripts

w = wall condition
∞ = freestream outside of boundary layer
0 = total condition
1 = injected substance
2 = original substance in boundary layer
i = ith species, incompressible

Received September 9, 1963 This work was accomplished under Contract AF 04(694)-1 and appears in more comprehensive form in Space Technology Laboratories Report 6130-6357-KU000

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EXTENSIVE uses of ablation to protect bodies in high-speed flight have focused attention on the effect on the boundary layer of mass injection More recently the problem of radiation has created an interest in quenching the high temperatures in a boundary layer by this mass injection

In examining the laminar boundary layer equations for a mixture as given, for example, in Ref 1 for the assumptions of Prandtl and Lewis numbers = 1, and without pressure gradient or chemical reaction the continuity for a single species, momentum and energy equations, respectively, can be written as follows:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial C_i}{\partial y}\right) \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho u \frac{\partial h_0}{\partial x} + \rho v \frac{\partial h_0}{\partial y} = \frac{\partial}{\partial y}\left(\mu \frac{\partial h_0}{\partial y}\right) \tag{3}$$

Many authors have indicated the solution to Eq (3) can be written in terms of the solution to Eq (2) as follows:

$$h + u^2/2 = (h_{0\infty} - h_w)u/u_\infty + h_\infty \tag{4}$$

with the boundary conditions

$$y = 0 \qquad u = 0 \qquad h = h_w \qquad C_1 = C_{1w}$$

$$y = \delta \qquad u = u_\infty \qquad h = h_\infty \qquad C_1 = 0$$

Likewise, the relation between C and u can be obtained

$$C_1/C_{1w} = 1 - u/u_\infty \tag{5}$$

Eliminating u/u_∞ between Eqs (4) and (5) provides the corresponding relation between h and C,

$$h = h_w(C_1/C_{1w}) + (1 - C_1/C_{1w})(h_\infty + C_1/C_{1w}u_w^2/2) \tag{6}$$

Differentiating either Eq (4) or (6) yields h maximum

$$\frac{h_{\max}}{h_{0\,\infty}} = \frac{1}{4}\left[1 + 2\left(\frac{h_w}{h_{0\,\infty}}\right) + \left(\frac{h_w}{h_{0\,\infty}}\right)^2\right] \text{ for } \frac{h_\infty}{h_{0\,\infty}} \ll 1 \tag{7}$$

For a cold wall $h_w/h_{0\infty} \ll 1$, Eq (7) further reduces to $h_{\max}/h_{0\infty} \simeq \frac{1}{4}$ as has been noted in Ref 1, but it is noted here that the result is independent of both velocity and concentration profiles as well as the amount and substance being injected The temperature, however, does depend on both the con-

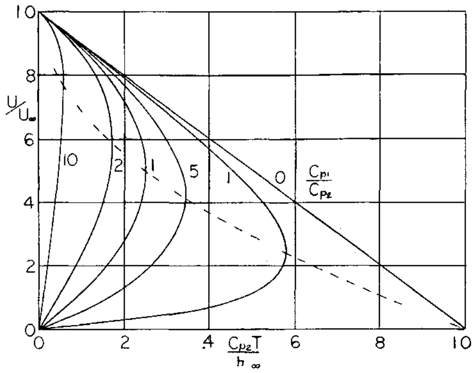

Fig 1 Boundary layer temperature profiles with injection, $h_\infty/h_{0\infty} \ll 1$, $h_w/h_{0\infty} \ll 1$, $h_{0\infty} = h_\infty + u_\infty^2/2g \simeq u_\infty^2/2g$, $Pr = Le = 1$, (1) → injected fluid, (2) → original fluid

centration and specific heat of the injectant For a mixture limited to two species of constant specific heats

$$\langle C_p \rangle = \frac{C_1 C_{p1} + C_2 C_{p2}}{C_1 + C_2} = C_{p2}\left[C_1\left(\frac{C_{p1}}{C_{p2}} - 1\right) + 1\right]$$

since $C_1 + C_2 = 1$ Introduction of this expression into Eq (7) gives the following expression for the temperature in terms of concentration:

$$T = h/\langle C_p \rangle = \frac{h_w\, C_1/C_{1w} + (1 - C_1/C_{1w})(h_\infty + C_1/C_{1w}\, u_\infty^2/2)}{C_{p2}[C_1(C_{p1}/C_{p2} - 1) + 1]} \quad (8)$$

which after rearrangement consistent with the cold wall and high-velocity approximations becomes

$$\frac{TC_{p2}}{h_{0\infty}} = \frac{C_1/C_{1w}(1 - C_1/C_{1w})}{C_1[(C_{p1}/C_{p2}) - 1] + 1} \quad (9)$$

In Eq (9) the wall concentration cannot be specified directly and, in general, differs from one

Considering now the boundary layer velocity; this is composed of convective and diffusive parts where for a two-component mixture the convective part at the wall can be expressed as in Ref 2:

$$v_w = \frac{D}{C_{2w}} \frac{\partial C_2}{\partial y}\bigg|_w \quad (10)$$

Considering Eq (5) this becomes

$$v_w = \frac{-D}{(1 - C_{1w})} \frac{C_{1w}}{u_\infty} \frac{\partial u}{\partial y}\bigg|_w \quad (11)$$

Both Pr and Le have been taken as 1; therefore the Schmidt number Sc is also 1, which produces the relation $\mu = \rho D$ to introduce into Eq (11):

$$v_w = \frac{C_{1w}}{\rho_w(1 - C_{1w})u_\infty} \mu_w \frac{\partial u}{\partial y}\bigg|_w = \frac{C_{1w}}{\rho_w(1 - C_{1w})} \frac{\tau_w}{u_\infty} \quad (12)$$

The mass of the injected fluid is $\dot{m}_1 = \rho_{1w} v_{1w} = \rho_w C_{1w} v_{1w}$, where v_{1w} again has a convective and diffusive velocity By using Eq (12) for the convective velocity, the fact that $Sc = 1$, and Eq (5),

$$\dot{m}_1 = \frac{C_{1w}\mu_w}{(1 - C_{1w})u_\infty} \frac{\partial u}{\partial y}\bigg|_w = \frac{C_{1w}}{(1 - C_{1w})} \frac{\tau_w}{u_\infty} \quad (13)$$

and C_{1w} then may be expressed in familiar parameters evaluated at the wall:

$$C_{1w} = \frac{1}{1 + \tau_w/\dot{m}_1 u_\infty} \quad (14)$$

In general τ_w decreases with increasing injection rates, even becoming 0 at a finite rate; therefore $C_{1w} \to 1$ as the limiting case for injection as the boundary layer is separated from the surface Taking $C_{1w} = 1$, Eq (9) can be used to show the effect of C_{p_1} on the temperature through the boundary layer This is shown in Fig 1 For constant properties and $C_{p_1} = C_{p_2}$, $(C_{p_2}T/h_{0\infty}) = \frac{1}{4}$, as might be guessed from Eq (7) Increasing C_{p_2}/C_{p_1} from 1 lowers the maximum $C_{p_2}T/h_{0\infty}$ and appears to move it away from the wall, while the inverse is true for $C_{p_2}/C_{p_1} < 1$ Rearrangement of the terms of the injection parameter $\tau_w/\dot{m}_1 u_\infty$ gives the following:

$$\frac{\tau_w}{\dot{m}_1\, u_\infty} = \frac{C_f/2\; Re_\infty^{1/2}}{(\dot{m}_1/\rho_\infty u_\infty) Re_\infty^{1/2}} \quad (15)$$

which, for constant density, reduces to the same form as Ref 2:

$$\frac{\tau_w}{\dot{m}_1\, u_\infty} = \frac{C_f/2\; Re_\infty^{1/2}}{v_w/u_\infty\; Re_\infty^{1/2}} \quad (15a)$$

In this form all terms can be evaluated from Ref 2, including some effects of T_w/T_∞, but in order to include the effects of compressibility it is possible to turn to Ref 3 where the method of Chapman and Rubesin was used to solve the compressible boundary layer equations with a finite normal velocity at the wall The convective part of v_{1w}, as expressed from Eq (10), can be combined with the diffusive part so that $\dot{m}_1 = \rho_w v_w$ which, in terms of the parameters evaluated in Ref 3, becomes

$$\dot{m}_1 = \rho_w v_w = -\frac{\rho_\infty u_\infty f(0)}{2}\left(\frac{C'}{Re_\infty}\right)^{1/2} \quad (16)$$

Very conveniently, a corresponding expression exists for the skin friction:

$$C_f = \frac{f''(0)}{2}\left(\frac{C'}{Re_\infty}\right)^{1/2} \quad (17)$$

By use of Eqs (16, 17, and 15a), C_{1w}, by means of Eq (14), can be evaluated as follows:

$$C_{1w} = \frac{1}{1 - \frac{1}{2} f''(0)/f(0)} \quad (18)$$

Values for $f(0)$ and $f''(0)$ for certain values of injection have been taken directly from Ref 3 and are given with C_{1w} in Table 1 After having thus expressed the wall concentration, Eq (7) relates the temperature parameter $C_{p_2}T/h_{0\infty}$ to a group of parameters determined by the amount and properties of the injected gas Differentiating Eq (9) yields C_1^* for $C_{p_2} T_{\max}/h_{0\infty}$:

$$C_1^{*2} + \frac{2}{(C_{p1}/C_{p2} - 1)} C_1^* - \frac{C_{1w}}{(C_{p1}/C_{p2} - 1)} = 0 \quad (19)$$

Figure 2 is a plot of C_1^* which, when used with Eq (9), gives $C_{p_2}T_{\max}/h_{0\infty}$ for various wall concentrations with only the injected gas specific heat as a parameter This is shown in Fig 3, where one can see, at least for $C_{p_1}/C_{p_2} \geqslant \frac{1}{2}$, that most of the change in $C_{p_2}T_{\max}/h_{0\infty}$ has taken place by the time a C_{1w} of about 0 8 is obtained

It is particularly interesting to note that the Chapman-Rubesin parameter $C' = T_\infty\mu/T\mu_\infty = \rho\mu/\rho_\infty\mu_\infty$ enters into Eqs (16) and (17) in the same manner; hence it cancels in the injection parameter of Eq (18) This implies that a vari-

Table 1 Blasius solutions with injection at the wall

$f(0)$	0	−0 5	−0 75	−1 00	−1 21 (est)
$f''(0)$	1 3282	0 6580	0 3745	0 1421	0
C_{1w}	0	0 60	0 80	0 93	1

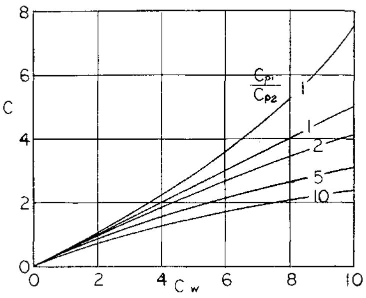

Fig 2 Concentration at boundary layer maximum temperature, $h_\infty/h_{0\infty} \ll 1$, $h_w/h_{0\infty} \ll 1$, $h_{0\infty} = h_\infty + u_\infty^2/2g \simeq u_\infty^2/2g$, $Pr = Le = 1$, (1) → injected fluid, (2) → original fluid

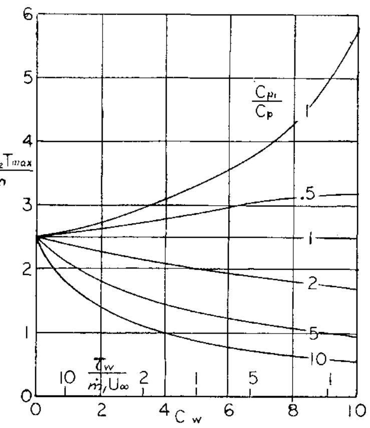

Fig 3 Maximum boundary layer temperature with injection, $h_\infty/h_{0\infty} \ll 1$, $h_w/h_{0\infty} \ll 1$, $h_{0\infty} = h_\infty + u_\infty^2/2g \simeq u_\infty^2/2g$, $Pr = Le = 1$, (1) → injected fluid, (2) → original fluid

ation of $\rho\mu$ has the same influence on injection as on skin friction under the simplifying assumptions, and Eq (18) is independent of the gas being injected and the wall temperature The solution in Ref 3, being simply that of Chapman and Rubesin with a normal velocity component at the wall, does not consider any interaction between the injected and original gases, thus implying that transport properties are similar In Refs 1 and 4, the authors have considered boundary layers in which there is a large variation of $\rho\mu$ A large number of gases viscosities are within a factor of 2 or so; therefore, $\rho\mu$ varies within an order of magnitude as density or molecular weight For an effective average value of C' the "reference enthalpy" method has been used, and, at least for the stagnation point solutions, the following relation has proved adequate:

$$\langle c' \rangle = (\rho_w\mu_w/\rho_w\mu_\infty)^{0\ 2} \tag{20}$$

In Ref 5 numerical results of air/air, CO_2/air, and He/air injection are presented The results seem to indicate that the simplified approach of this paper is surprisingly good for a gas such as CO_2, since a table for CO_2/air injection corresponding to Table 1 would differ very little Helium, on the other hand, behaves even more differently than one would expect from the fact that $\rho\mu$ for helium differs from $\rho\mu$ for air by about an order of magnitude For this reason the simplifications of $Le = Pr = Sc = 1$ may not adequately represent the case of a monatomic or light gas injected into the boundary layer

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Stratification in a Pressurized Container with Sidewall Heating

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A method to determine the temperature profile due to stratification in a pressurized tank containing a liquid and subjected to sidewall (aerodynamic) heating is described It is shown that a straightforward empirical approach yields satisfactory correlation with the available data

Nomenclature

A = tank wall heated area, ft²
c = constant, ft²
C_p = liquid specific heat, Btu/lb-°R
D = tank diameter, ft
I = ratio of heat absorbed in stratified layer to heat input at sidewall
Q = heat transfer rate through wall, Btu/sec
T = temperature, R
t = time, sec
y = distance below surface ft
ρ = density, lb/ft³

Subscripts

s = saturation
b = initial bulk temperature

Introduction

IN Ref 1, the importance of determining the degree of stratification occurring in a cryogenic propellant tank due to aerodynamic heating was discussed A method was developed to predict the depth of the stratified layer and the maximum temperature; however, no attempt was made to determine the temperature gradient in the stratified layer By use of the data obtained by the authors in Ref 1, as well as other data, it will be shown that a straightforward empirical approach yields satisfactory correlation with stratification temperature data obtained in tests

Discussion

The heat input from the wall of a tank will tend to warm the liquid near the wall and will cause a convective current to move up toward the surface When the warm liquid reaches the vicinity of the surface, it will spread toward the center of the tank and form a mechanically stable configuration with the warmest liquid at the surface Further, since the rising liquid leaves its energy source and, in effect, turns a corner, the stratified layer at the surface will have a different temperature profile than that which existed in the boundary layer at the wall

If the heat flux at the wall is low, the boundary layer that forms will move up the wall in the manner described in free convection theory If the heat flux is high, boiling will take place and, in most cases, the boiling will be in the nucleate range Since the tank is pressurized, the bulk liquid is effectively in a subcooled condition; the bubbles growing at the wall will collapse in the cooler boundary layer moving up the wall The net result is that the heat input at the wall produces a thin boundary layer at the wall which subsequently deposits hot liquid at the surface (Fig 1)

For a well-insulated tank, in which most of the heat input travels down from the top, it has been found[2] that the temperature profile is similar in shape to that obtained for a

Received September 11, 1963 The work reported in this paper was sponsored by the Space and Information Division of North American Aviation, Downey, Calif

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